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Joint Project "Machine Learning to Improve the Reliability of Complex Systems"

Statistical Analysis of the Efficiency of an Integrated Voltage Regulator by means of a Machine Learning Model Coupled with Kriging Regression

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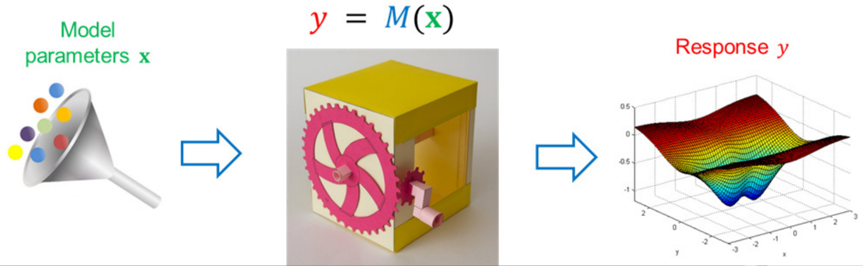
Statistical Analysis of the Efficiency of an Integrated Voltage Regulator by means of a Machine Learning Model Coupled with Kriging Regression – SPI 2019, 18-21 June, 2019

Intro: design process and needs

- ❑ Robust designs require assessment against uncertainty
- ❑ Optimize design to achieve better product performance

➢ Need for a **computational model**, *i.e. a procedure (e.g., analytical formula, algorithm, …) computing quantities of interest from input parameters*

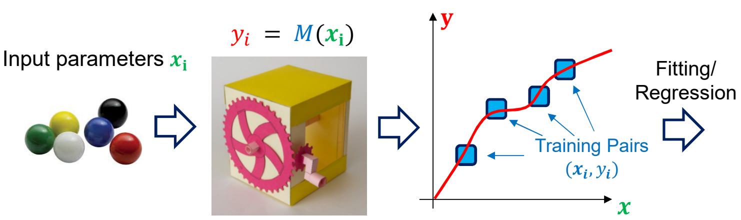


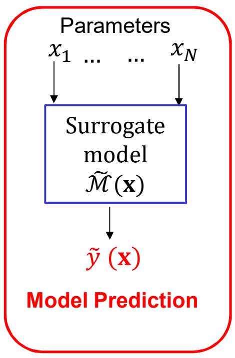
The crux of the matter

- **Computational models** are demanding in terms of memory and CPU time
- **Statistical simulations** are necessary for both
  - the assessment of the design robustness w.r.t. uncertainty and variability
  - the performance optimization

Surrogate Modeling

- A **surrogate model** $\tilde{\mathcal{M}}$ is an approximation of the **full-computational model** $\mathcal{M}$





- Determined from a <u>limited number of runs</u> of the full model
- Simpler **closed-form** relationship
- **Faster** then the full-model $\mathcal{M}$

Several **fitting techniques** are available

Least Squares Support Vector Machines (LS-SVM)

- **Machine learning** technique for both **classification** and **regression**
- **Kernel method** mapping data from the vector space to the feature space
- The # of unknowns is **independent** from **problem dimensionality (non-parametric regression)**

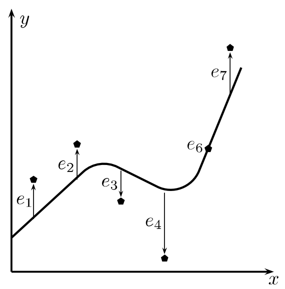
Generic formulation

$$\mathcal{M}(\mathbf{x}) = \sum_{i=1}^{L} \alpha_i K(\mathbf{x}_i, \mathbf{x}) + b$$

The **response** $y = \mathcal{M}_{LS-SVM}(x)$ is given as the **linear combination** of the kernel evaluated at the training samples x_i.

- Linear kernel : $K(\boldsymbol{x}, \boldsymbol{x}_i) = \boldsymbol{x}_i^T \boldsymbol{x}$,
- Polynomial kernel of degree d: $K(\boldsymbol{x}, \boldsymbol{x}_i) = \left(1 + \boldsymbol{x}_i^T \boldsymbol{x}/c\right)^d$,
- Radial basis function RBF kernel : $K(\boldsymbol{x}, \boldsymbol{x}_i) = \exp\left(-\|\boldsymbol{x} - \boldsymbol{x}_i\|^2/\sigma^2\right)$,
- MLP kernel : $K(\boldsymbol{x}, \boldsymbol{x}_i) = \tanh\left(k\, \boldsymbol{x}_i^T \boldsymbol{x} + \theta\right)$,

LS-SVM (cont'd)



$$\mathcal{M}_{LS-SVM}(\mathbf{x}) = \sum_{i=1}^{L} \alpha_i K(\mathbf{x}_i, \mathbf{x}) + b$$

Least squares solution

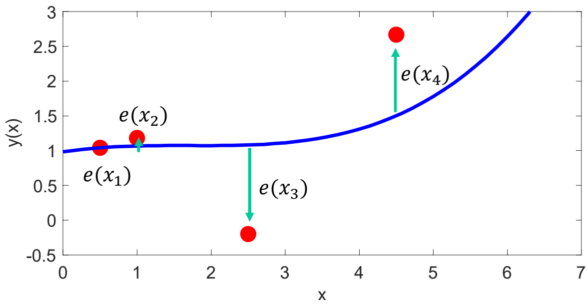
$$\begin{bmatrix} 0 & 1 & \dots & 1 \\ 1 & K(x_1, x_1) + \frac{1}{\gamma} & & K(x_1, x_L) \\ \vdots & & \ddots & \\ 1 & K(x_L, x_1) & & K(x_L, x_L) + \frac{1}{\gamma} \end{bmatrix} \begin{bmatrix} b \\ \alpha_1 \\ \vdots \\ \alpha_L \end{bmatrix} = \begin{bmatrix} 0 \\ y_1 \\ \vdots \\ y_L \end{bmatrix}$$

The coefficients α and b are estimated by solving a **simple linear system**

How accurate is the model prediction ???

Deterministic Regression & Error Function

- Training samples $\{(x_1, y_1), \ldots, (x_4, y_4)\}$



- Build a **surrogate mode** via a **deterministic regression**

$$y_i = M_{LS-SVM}(x_i) + e(x_i)$$

Surrogate Model — Model Error

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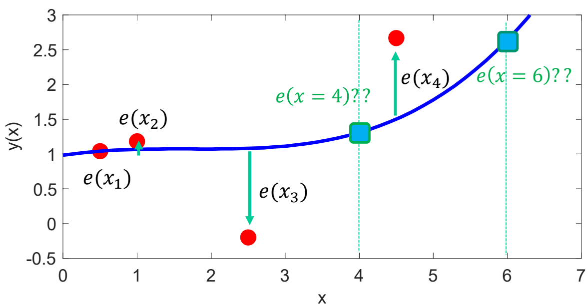

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Surrogate Model — Model Error

- Model Prediction

$$y(4) \approx M_{LS-SVM}(4) + ??$$
$$y(6) \approx M_{LS-SVM}(6) + ??$$

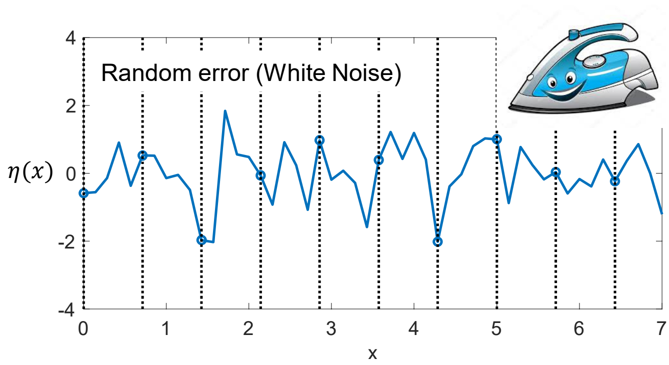
the regression error $e(x)$ is **only known** on the **training samples** x_i

How accurate is the prediction?
Confidence bounds are needed!!!

Probabilistic Interpretation

We can think of the error function $e(x)$ **in a probabilistic sense**

In practice, error is a smooth function

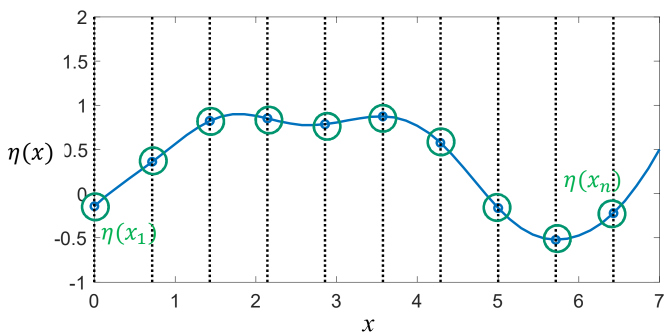


Probabilistic Interpretation

We can think of the error function $e(x)$ **in a probabilistic sense**

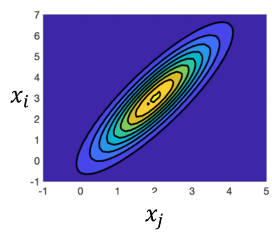
Assumption:

$e(x)$ is **smooth** → The values $\eta(x_1), .., \eta(x_n)$ **are correlated**



Correlation via Covariance function, e.g.,

$$k(x_i, x_j) = \sigma_f^2 \exp\left(-\frac{(x_i - x_j)^2}{2\sigma_l^2}\right)$$



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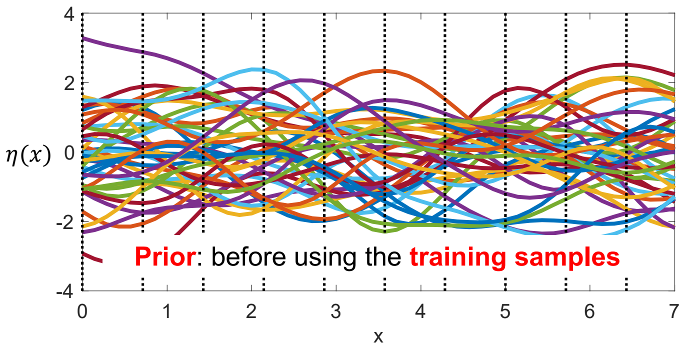

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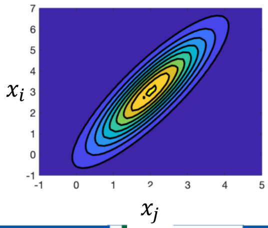
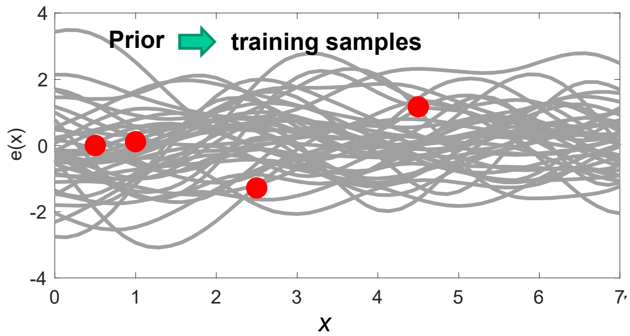


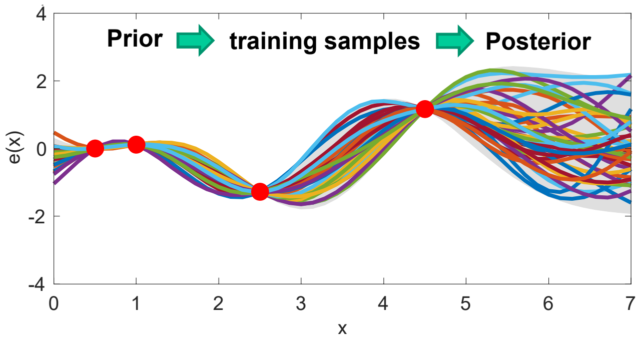
Gaussian Process (GP) Regression: from Prior to Posterior

- The value of the **error** $e(x_i)$ is known on the **training samples** $e(x_i)$



GP Regression: from Prior to Posterior

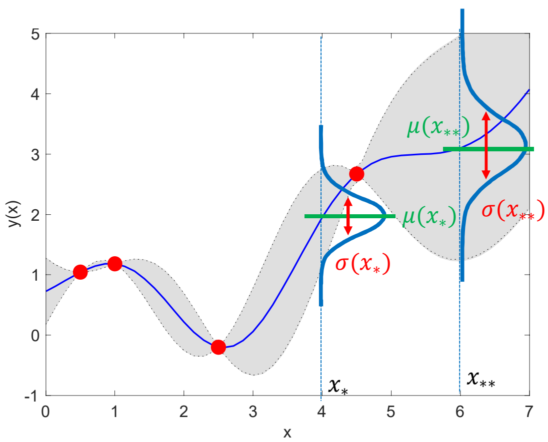
- The value of the **error** $e(x_i)$ is known on the **training samples** $e(x_i)$
- Discarding all the functions not compatible with the training samples



We consider **only all functions fitting our data**
Such functions can be described in terms of a **probability law** →
probabilistic model

GP Regression & Probabilistic Model

- The **resulting model** combines the **deterministic regression** M_{LS-SVM} with a **probabilistic model** of the error function $e(x)$



The model provides for any value x_* a **distribution** [3]

$$M(x_*) \sim N(\mu(x_*), \sigma^2(x_*))$$

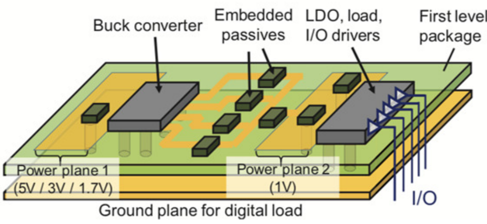
most probable value of the error | **confidence bounds**

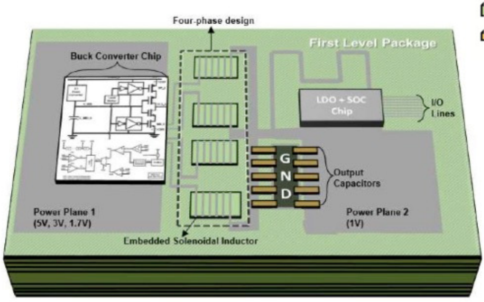
GP allows converting any **regression-based deterministic** model into a **probabilistic** one

[3] C. E. Rasmussen and C. K. I. Williams, Gaussian Processes for Machine Learning, MIT Press. Cambridge, Massachusetts, 2006.

Integrated Voltage Regulator (IVR)

- **System-in-package** (SiP) solution with buck converter, low-dropout/load and an **integrated inductor** on an organic package [4]



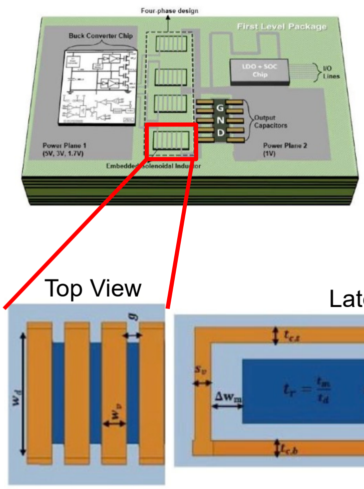


- The **buck converter** is used to drop the voltages of the power plane 1 to the level required by the power plane 2

The converter **efficiency** depends on the **integrated inductor**

[4] H. M. Torun, et al., "A Global Bayesian Optimization Algorithm and Its Application to Integrated System Design," in *IEEE Trans. Very Large Scale Integr. (VLSI) Syst.*, vol. 26, no. 4, pp. 792–802, April 2018.

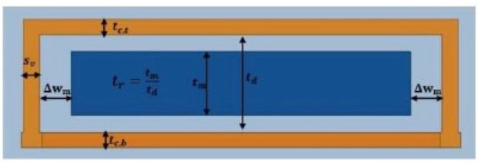
Embedded Solenoidal Inductor [5]



CONTROL PARAMETERS OF SOLENOIDAL INDUCTOR

Uniform random variables		Unit	$\mathcal{U}[Min; Max]$
Gap between windings	g	mil	$\mathcal{U}[4;6]$
Size of via	s_v	μm	$\mathcal{U}[80;120]$
Copper Trace Width	w_v	mil	$\mathcal{U}[9;11]$
Copper Thickness Bottom	$t_{c,b}$	μm	$\mathcal{U}[64;96]$
Copper Thickness Top	$t_{c,t}$	μm	$\mathcal{U}[64;96]$
Dielectric Thickness	t_d	μm	$\mathcal{U}[180;220]$
Dielectric Width	w_d	mil	$\mathcal{U}[59.4;60.6]$
Magnetic Core Width offset	Δw_m	mil	$\mathcal{U}[9;11]$

Lateral View



The IVR efficiency has been investigated by **considering 8 uniformly distributed parameters**

[5] R. Trinchero, at al., "Machine Learning and Uncertainty Quantification for Surrogate Models of Integrated Devices With a Large Number of Parameters," in IEEE Access, vol. 7, pp. 4056-4066, 2019.

IVR Results

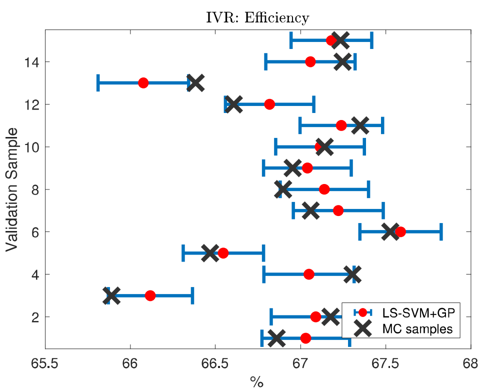
- **A subset of $L = 200$ training samples** is selected via Latin Hypercube Sampling
- The surrogate model predictions are **compared** with the results of a **MC simulation** with 10000 samples

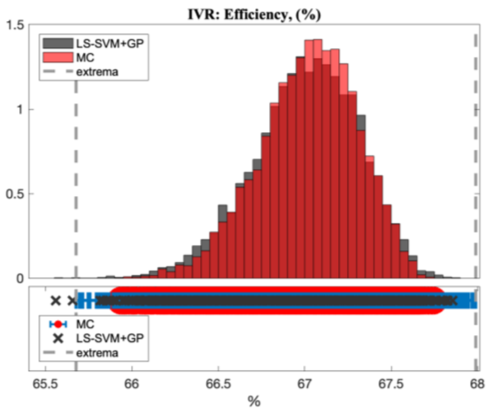
Method	Kernel Regression	RMSE	$\hat{\mu}$	$\hat{\sigma}$	t_{model}	t_{cost}
MC	–	–	67.01	0.31	–	7 days
LS-SVM	Linear	0.158	67.03	0.28	<1s	<1s
	Poly Order 2	0.166	67.04	0.29	<1s	<1s
	Poly Order 3	0.443	67.00	0.54	<1s	<1s
	RBF	0.152	67.02	0.28	1.4s	<1s
GP	–	0.162	67.02	0.28	<1s	2.4s
LS-SVM+GP	Linear	0.162	67.02	0.28	<1s	<1s
	RBF	0.156	67.02	0.28	1.5s	<1s
Sparse PC	Poly Order 9	0.170	67.02	0.28	<1s	<1s

LS-SVM and LS-SVM+GP regression with RBF kernel provides the **most accurate** metamodel

IVR Results (cont'd)

- The prediction and 95% **confidence intervals** estimated by the GP+LS-SVM (RBF) are compared with the results of a **MC simulation** with 10000 samples.





Excellent agreement for both the model prediction and Confidence Intervals

Conclusions

- **Surrogates** based **on Machine Learning regressions** represent effective solution for the UQ in nonlinear problems

- **Surrogates** are built from a **limited set of training samples** provided by the full-model

- **Gaussian Process regression (aka Kriging)** allows building accurate probabilistic models providing an estimation of the **model output** + **confidence bounds**

- FUTURE WORK**:** What is the limit in term of number of parameters?