A Hierarchical Approach to the Stochastic Analysis of Transmission Lines via Polynomial Chaos

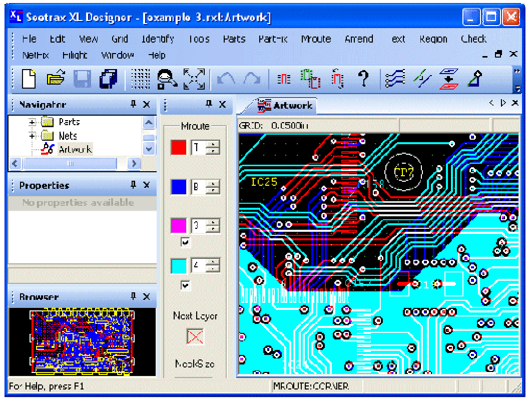
P. Manfredi and R. Trinchero

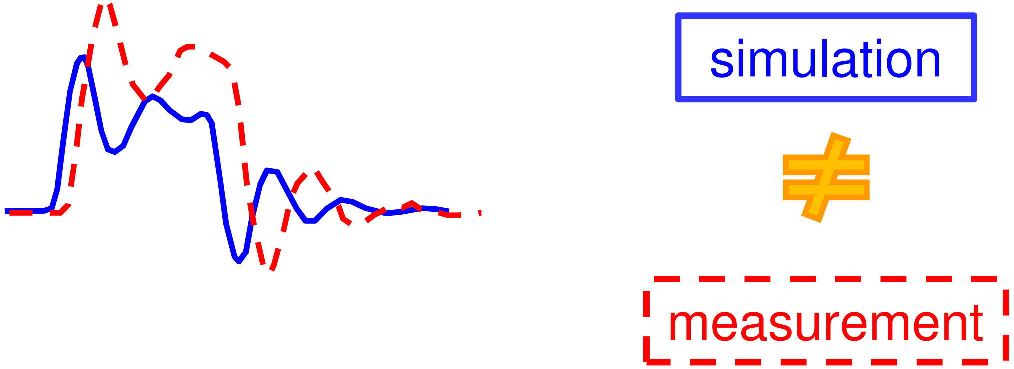
`paolo.manfredi@polito.it`

POLITECNICO DI TORINO
Dipartimento di Elettronica e Telecomunicazioni

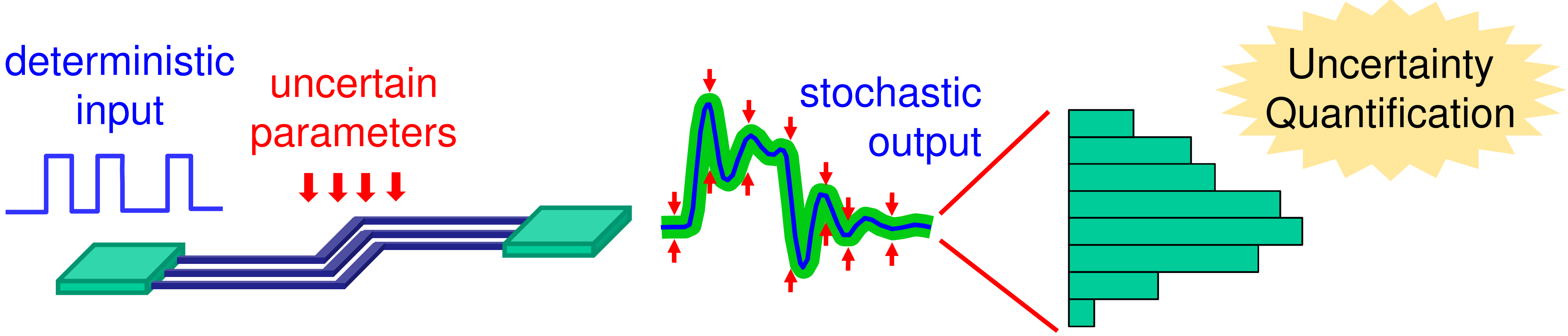
Manufacturing tolerances

- ☐ Predicted and measured responses can differ because of manufacturing variability, regardless of model accuracy





Polynomial chaos expansion (PCE)



- ☐ Uncertain parameters are collectively denoted as $\boldsymbol{\xi}$

- ☐ Stochastic responses are modeled as expansions of polynomials orthonormal to the distribution of $\boldsymbol{\xi}$ [1]

$$v(t,\boldsymbol{\xi}) \approx \sum_k v_k(t)\varphi_k(\boldsymbol{\xi})$$

deterministic coefficients

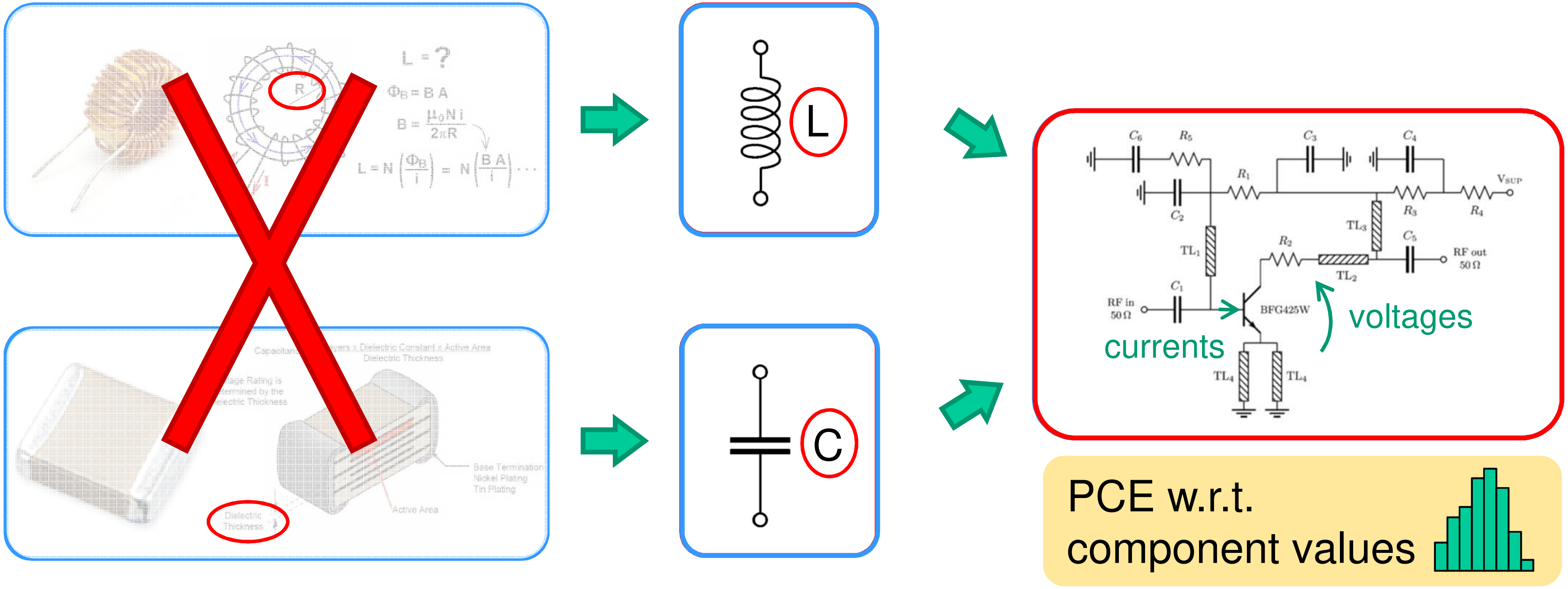
orthonormal polynomials

Calculation of PCE coefficients typically much faster than Monte Carlo

[1] P. Manfredi, D. Vande Ginste, I. S. Stievano, D. De Zutter, and F.G. Canavero, “Stochastic transmission line analysis via polynomial chaos methods: an overview,” IEEE Electromagn. Compat. Mag. (2017).

Simulation flow for lumped circuits

Physical level | Component level | Electrical output level



PCE w.r.t. component values

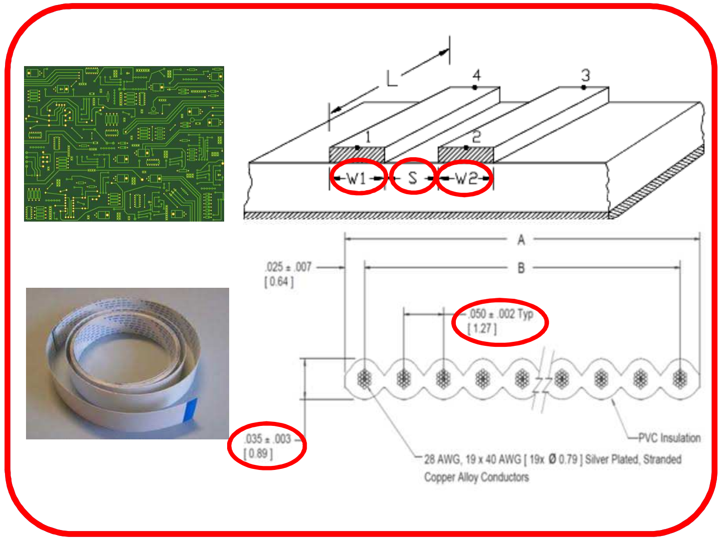
Geometry, materials
Distribution:
standard (e.g., Gaussian)

Component values
Distribution: standard or
non-standard (but still <u>independent</u>)

- ☐ Variability is usually modeled at component level

Simulation flow for distributed transmission lines

Physical (low) level



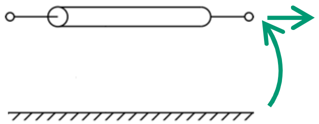
variability

Component (mid) level

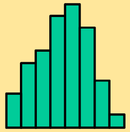
per-unit-length
RLGC
parameters

Electrical (high) output level

terminal
voltages / currents



PCE w.r.t.
physical params



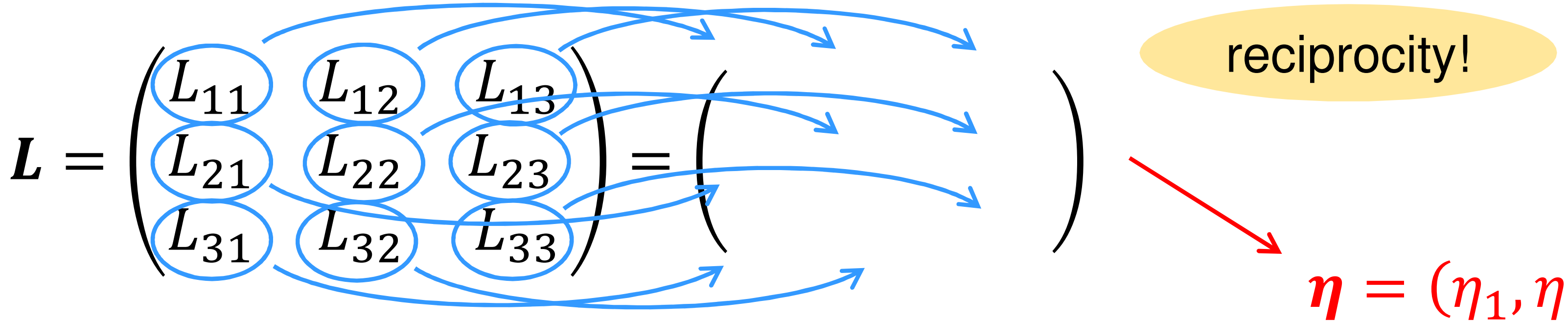
- ☐ Variability is modeled at physical level because per-unit-length parameters are NOT independent!

Modeling at
component
level possible?

Step #0: identification of random variables

- ❑ A hierarchical approach is implemented, in which the new random variables are (mid-level) entries of per-unit-length matrices [2]

- ❑ A different variable is assigned to each distinct entry:



$$\boldsymbol{L} = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

reciprocity!

$$\boldsymbol{\eta} = (\eta_1, \eta_2, \dots)$$

$$\boldsymbol{C} = \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix} = \begin{pmatrix} \eta_7 & \eta_8 & \eta_9 \\ \eta_8 & \eta_{10} & \eta_{11} \\ \eta_9 & \eta_{11} & \eta_{12} \end{pmatrix}$$

- ❑ Other properties/assumptions (e.g., PEC planes, shields, homogeneity) may lead to further reduction in the number of mid-level parameters $\boldsymbol{\eta}$

[2] P. Manfredi, “A hierarchical approach to dimensionality reduction and nonparametric problems in the polynomial chaos simulation of transmission lines,” IEEE Trans. Electromagn. Compat. (early access)

Advantages

1. Dimensionality reduction when per-unit-length (mid-level) parameters are fewer than physical (low-level) parameters

   Cardinality: $|\boldsymbol{\eta}| < |\boldsymbol{\xi}|$

2. Can deal with nonparametric problems, for which low-level parameters cannot be explicitly defined

1:1 corr
between
per-unit-le

3. Can achieve higher accuracy for a given expansion order

$$v(\cancel{f(\boldsymbol{\xi})}) \approx \sum_k v_k \varphi_k(\boldsymbol{\xi}) \Rightarrow v(\boldsymbol{\eta}) \approx \sum_k \tilde{v}_k \tilde{\varphi}_k(\boldsymbol{\eta})$$

$$\downarrow$$

per-unit-length parameters $\boldsymbol{\eta} = f(\boldsymbol{\xi})$

classical PCE

hierarchical PCE

Step #1: mixture of Gaussians (MoG) fit

- ❑ Empirical distribution of per-unit-length parameters (dependent entries $\boldsymbol{\eta}$) is fitted using a mixture of Gaussians (MoG)

- ❑ A MoG is a weighed combination of correlated Gaussian distributions

weights

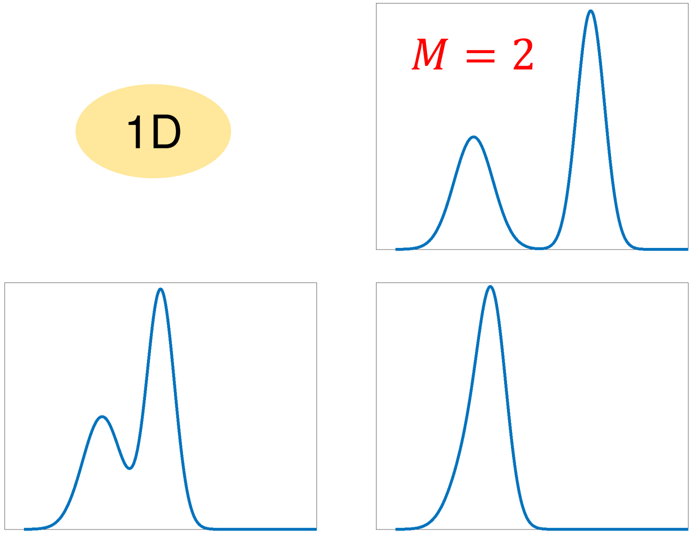
means

$$\rho(\boldsymbol{\eta}) = \sum_{m=1}^{M} w_m \frac{e^{-\frac{1}{2}(\boldsymbol{\eta}-\boldsymbol{\mu}_m)^T \boldsymbol{\Sigma}_m^{-1}(\boldsymbol{\eta}-\boldsymbol{\mu}_m)}}{\sqrt{\det(2\pi\boldsymbol{\Sigma}_m)}}$$

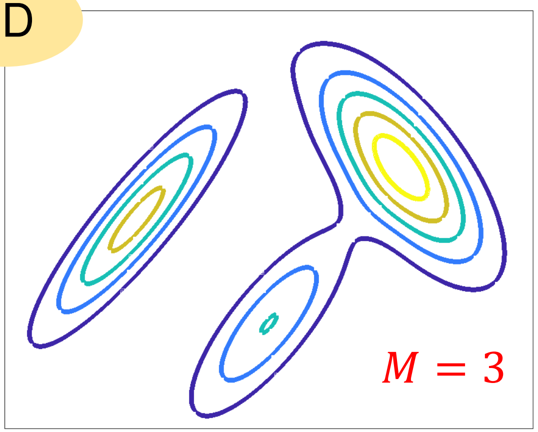
covariance matrices

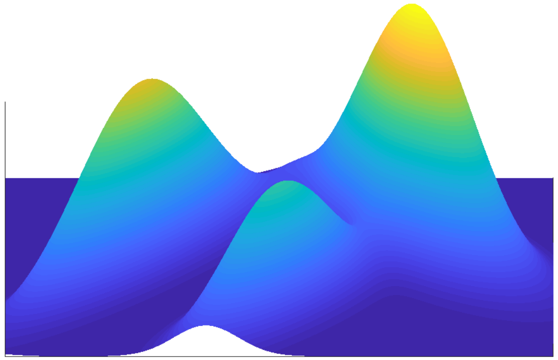
analytical model

1D



2D





Step #2: calculation of suitable basis functions

❑ Classical PCE uses standard polynomials (available a priori)

❑ Suitable basis function to be computed for correlated MoG distribution

❑ This is achieved through Gram-Schmidt orthogonalization [3]

Linearly independent monomials $\{\Psi_k\}$ ⇨

previously-computed polynomials
(iterative procedure)

$$\hat{\varphi}_k = \Psi_k - \sum_{j=1}^{k-1} \underbrace{\mathrm{E}\{\Psi_k \tilde{\varphi}_j\}} \tilde{\varphi}_j$$

(orthogonalization)

$$\tilde{\varphi}_k = \frac{\hat{\varphi}_k}{\sqrt{\underbrace{\mathrm{E}\{\hat{\varphi}_k^2\}}}}$$

(normalization)

expectations = scalar coefficients
(computed analytically or numerically)

[3] C. Cui and Z. Zhang, "Stochastic collocation with non-Gaussian correlated process variations: theory, algorithms and applications," IEEE Trans. Compon. Packag. Manuf. Technol. (early access).

Step #3: stochastic Galerkin method

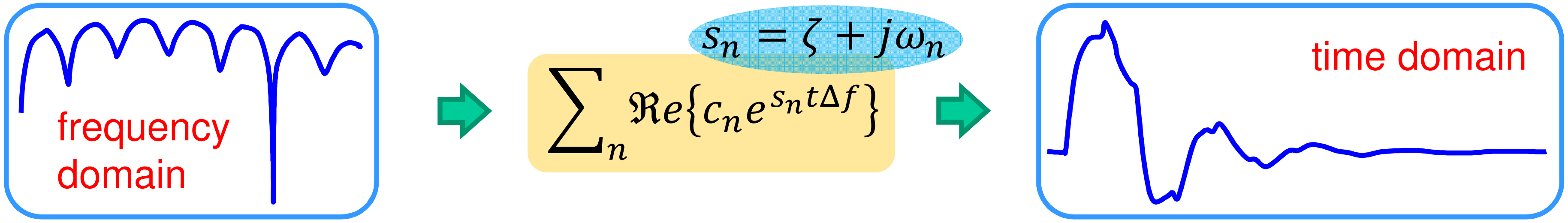
- ❑ Voltage and current PCE coefficients are computed via stochastic Galerkin method [1]

original, stochastic transmission line

Galerkin projection

augmented, deterministic transmission line

- ❑ To obtain transient results, the augmented transmission line can be:
  - ❖ Simulated directly into SPICE (lossless lines)
  - ❖ Solved in frequency domain and results post-processed with numerical inversion of Laplace transform (NILT) (lossy lines)

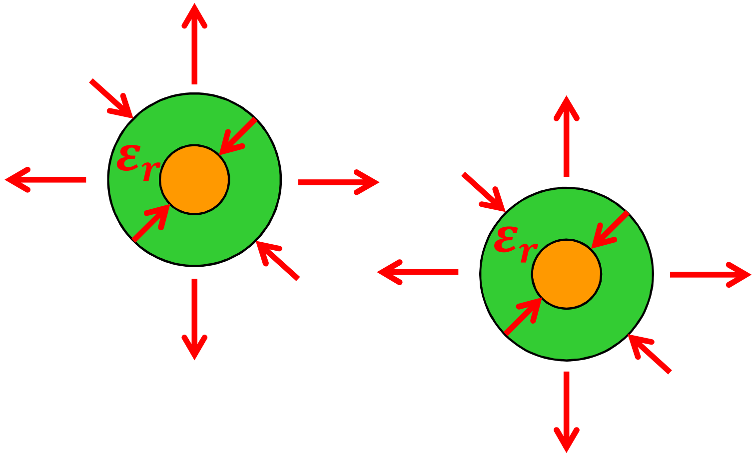


$$s_n = \zeta + j\omega_n$$

$$\sum_n \Re e\{c_n e^{s_n t \Delta f}\}$$

Application #1: two wires

❑ Two wires with random position, geometry, and material properties [2]

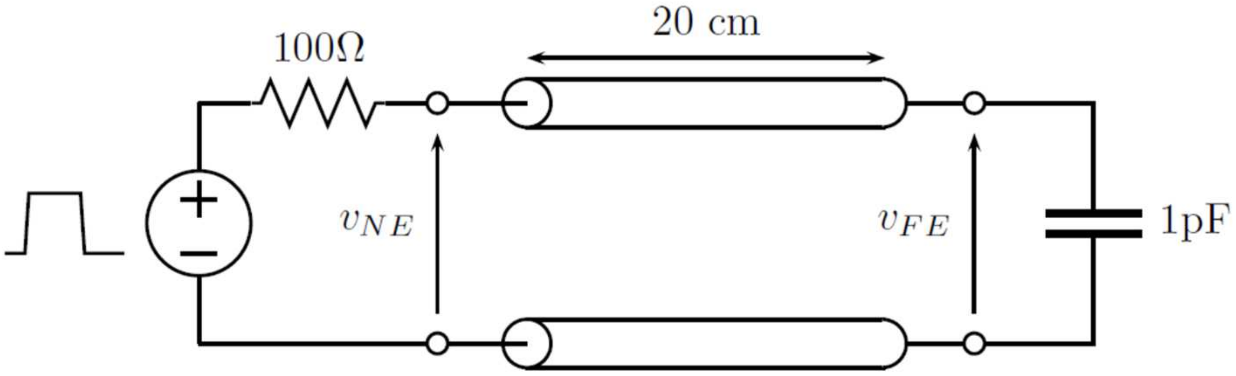


ξ

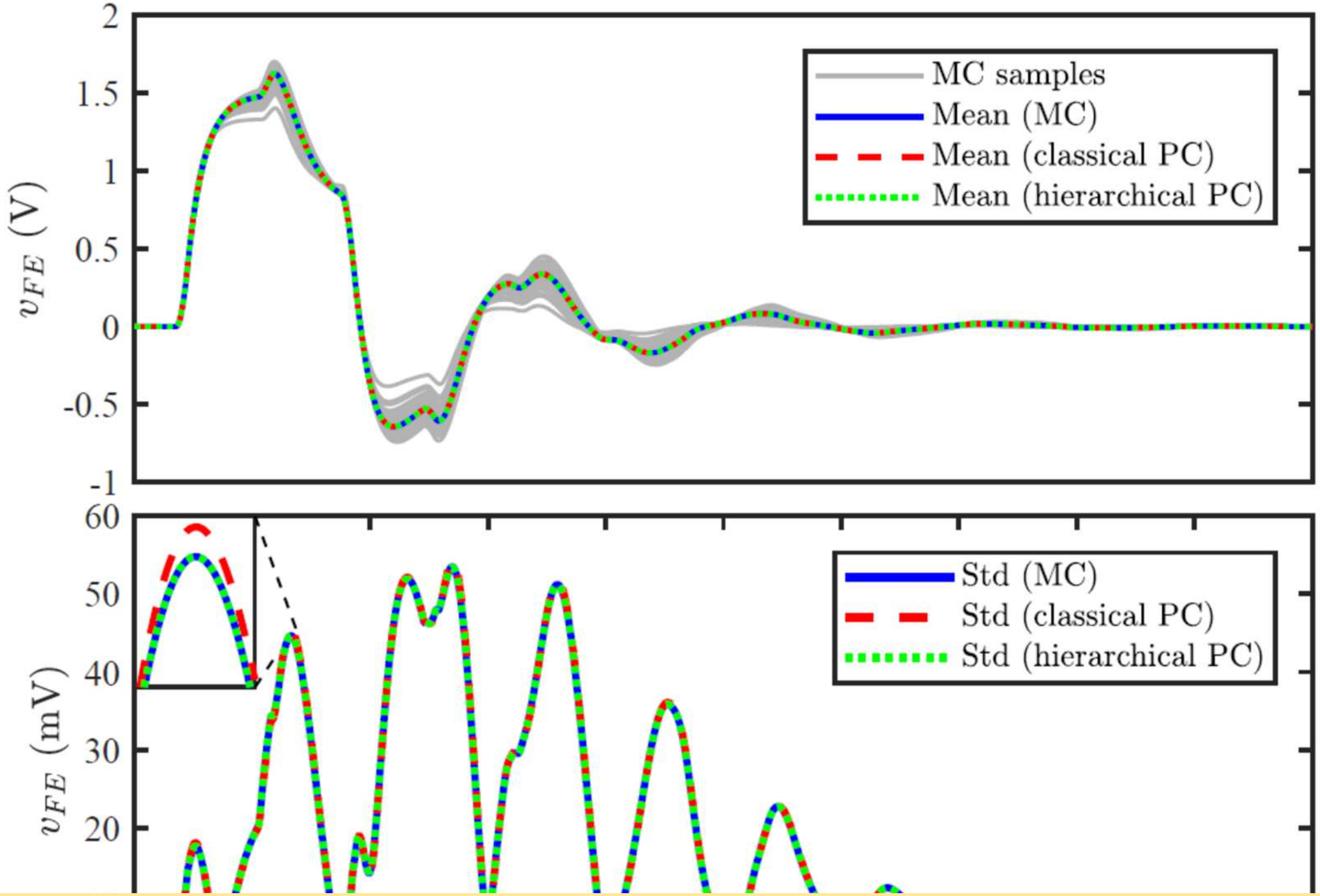
low-level (physical):
4 coordinates (x-y) [constrained to avoid overlap]
4 geometrical (wire and dielectric radii)
2 material (dielectric permittivity)
TOTAL = 10

η

mid-level (per-unit-length):
$L = \eta_1$
$C = \eta_2$
TOTAL = 2



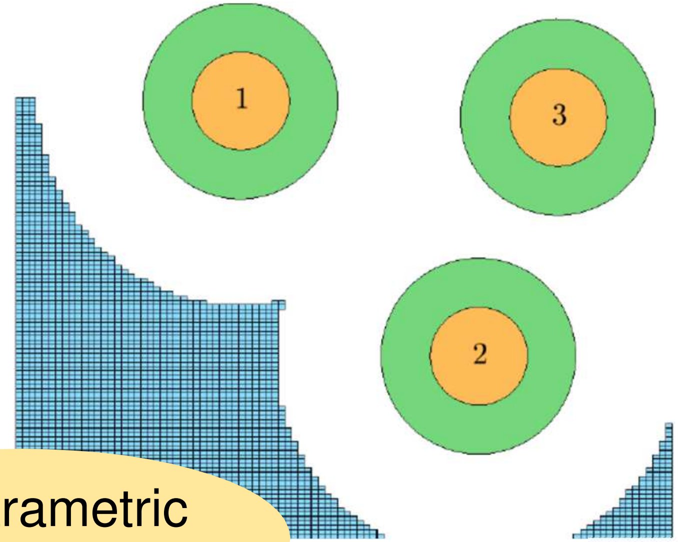
Application #1: results



Empirical distribution of per-unit-length parameters	(all methods):	48.9 s	
Basis functions	(hierarchical PC):	22.6 s	
SPICE simulation	(Monte Carlo):	3490.0 s	
	(classical PC):	357.0 s	(9.8x)
	(hierarchical PC):	0.5 s	(7200x)

Application #2: three wires above ground

- ☐ Three wires with completely random position (sequential placement) [2]

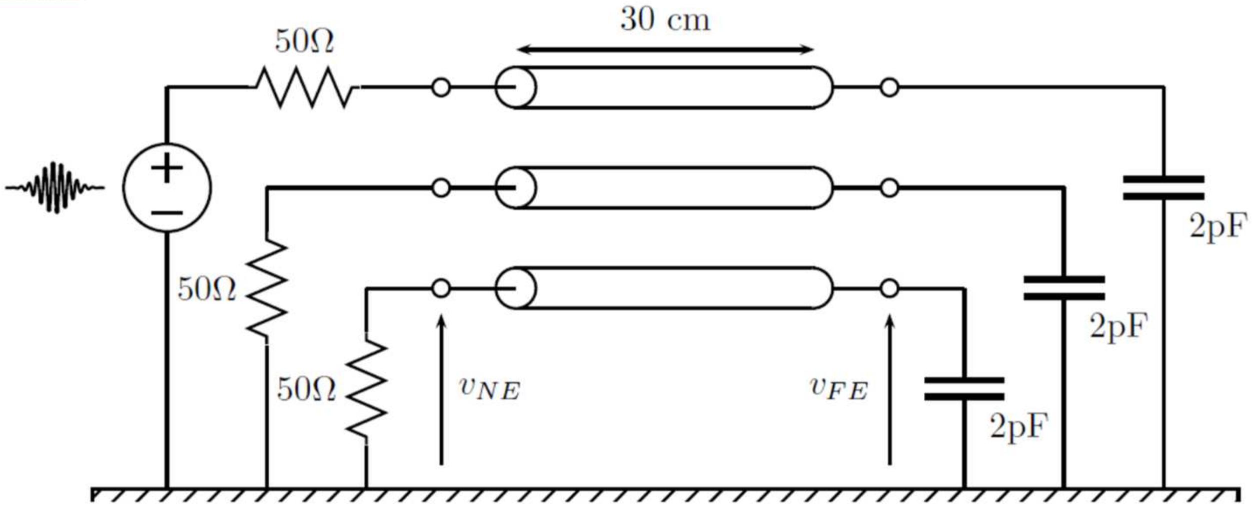


nonparametric problem!

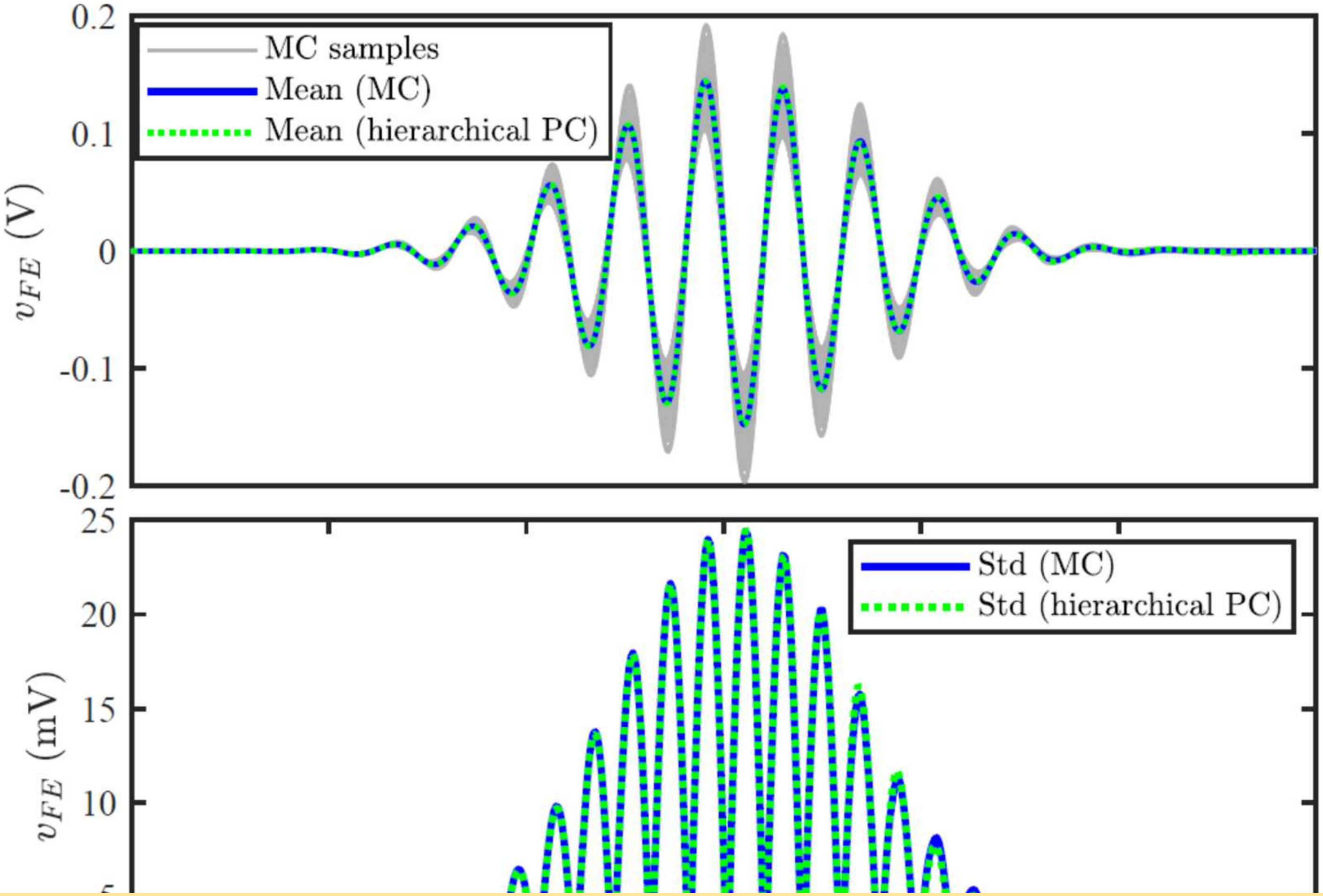
ξ low-level (physical): N/A

classical PCE N/A !

η mid-level (per-unit-length):
6 per-unit-length inductance entries
6 per-unit-length capacitance entries
TOTAL = 12



Application #2: results



Empirical distribution of per-unit-length parameters	(all methods):	226 s	
Basis functions	(hierarchical PC):	5 s	
SPICE simulation	(Monte Carlo):	4253 s	
	(classical PC):	N/A	
	(hierarchical PC):	23 s	(185x)

Conclusions

- ☐ Novel hierarchical approach for stochastic analysis of transmission lines
- ☐ PCE-based modeling w.r.t. mid-level (per-unit-length) parameters
- ☐ Empirical distribution of per-unit-length parameters fitted using a MoG
- ☐ Suitable basis functions computed via Gram-Schmidt orthogonalization
- ☐ PCE coefficients of line response obtained with Galerkin-based simulation
- ☐ Higher accuracy for a given expansion order
- ☐ Possible dimensionality reduction (⇒ higher efficiency)
- ☐ Handling of nonparametric problems (e.g., sequential wire placement)

References

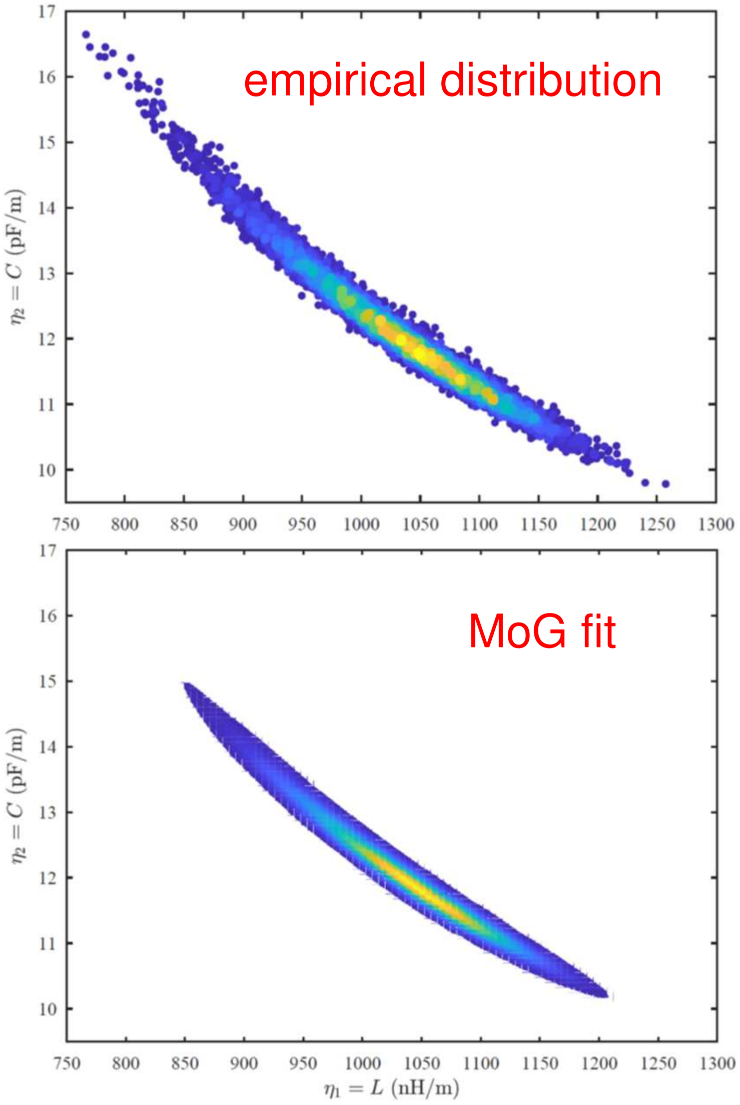
[1] P. Manfredi, D. Vande Ginste, I. S. Stievano, D. De Zutter, and F.G. Canavero, “Stochastic transmission line analysis via polynomial chaos methods: an overview,” IEEE Electromagn. Compat. Mag., vol. 6, no. 3, pp. 77–84, 2017.

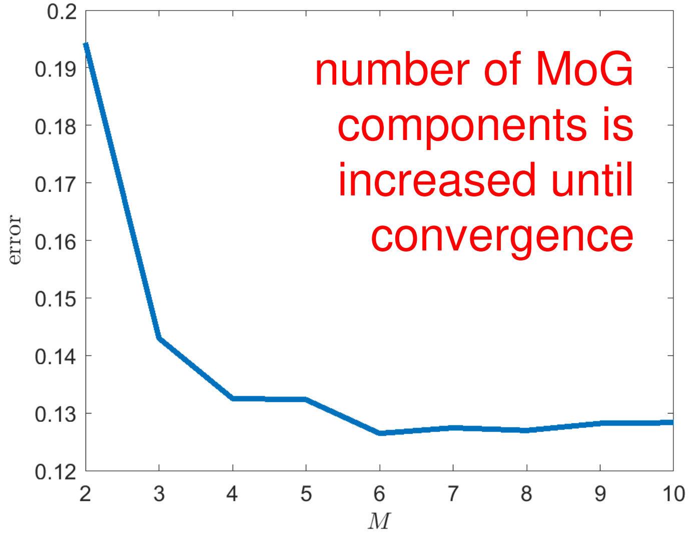
[2] P. Manfredi, “A hierarchical approach to dimensionality reduction and nonparametric problems in the polynomial chaos simulation of transmission lines,” IEEE Trans. Electromagn. Compat. (early access).

[3] C. Cui and Z. Zhang, “Stochastic collocation with non-Gaussian correlated process variations: theory, algorithms and applications,” IEEE Trans. Compon. Packag. Manuf. Technol. (early access).

Thank you for your attention!

Application #1: MoG fit & polynomial basis

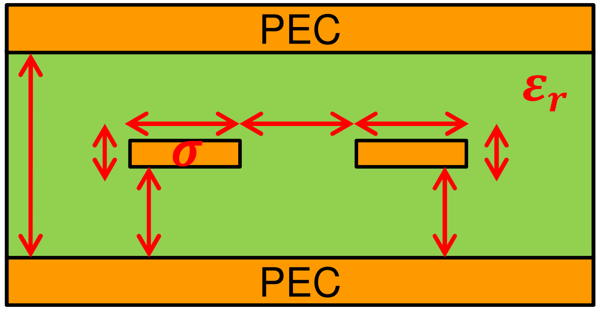




number of MoG components is increased until convergence

Application #3: lossy stripline

- ☐ Stripline interconnect with frequency-dependent conductor losses



$$\boldsymbol{Z}(s,\boldsymbol{\xi}) = \boldsymbol{R}_{dc}(\boldsymbol{\xi}) + \sqrt{s/\pi}\boldsymbol{R}_{hf}(\boldsymbol{\xi}) + s\boldsymbol{L}_{dc}(\boldsymbol{\xi})$$

SPICE model for frequency-dependent per-unit-length impedance matrix

PEC ground

$$\boldsymbol{R}_{dc} = \begin{pmatrix} \overset{\eta_1}{\boxed{R_{11}}} & \boxed{0} \\ \boxed{0} & \boxed{R_{22}} \end{pmatrix} \eta_2$$

$$\boldsymbol{R}_{hf} = \eta_3 \begin{pmatrix} \boxed{R'_{11}} & \boxed{R'_{m}} \\ \boxed{R'_{m}} & \underset{\eta_5}{\boxed{R'_{22}}} \end{pmatrix} \eta_4$$

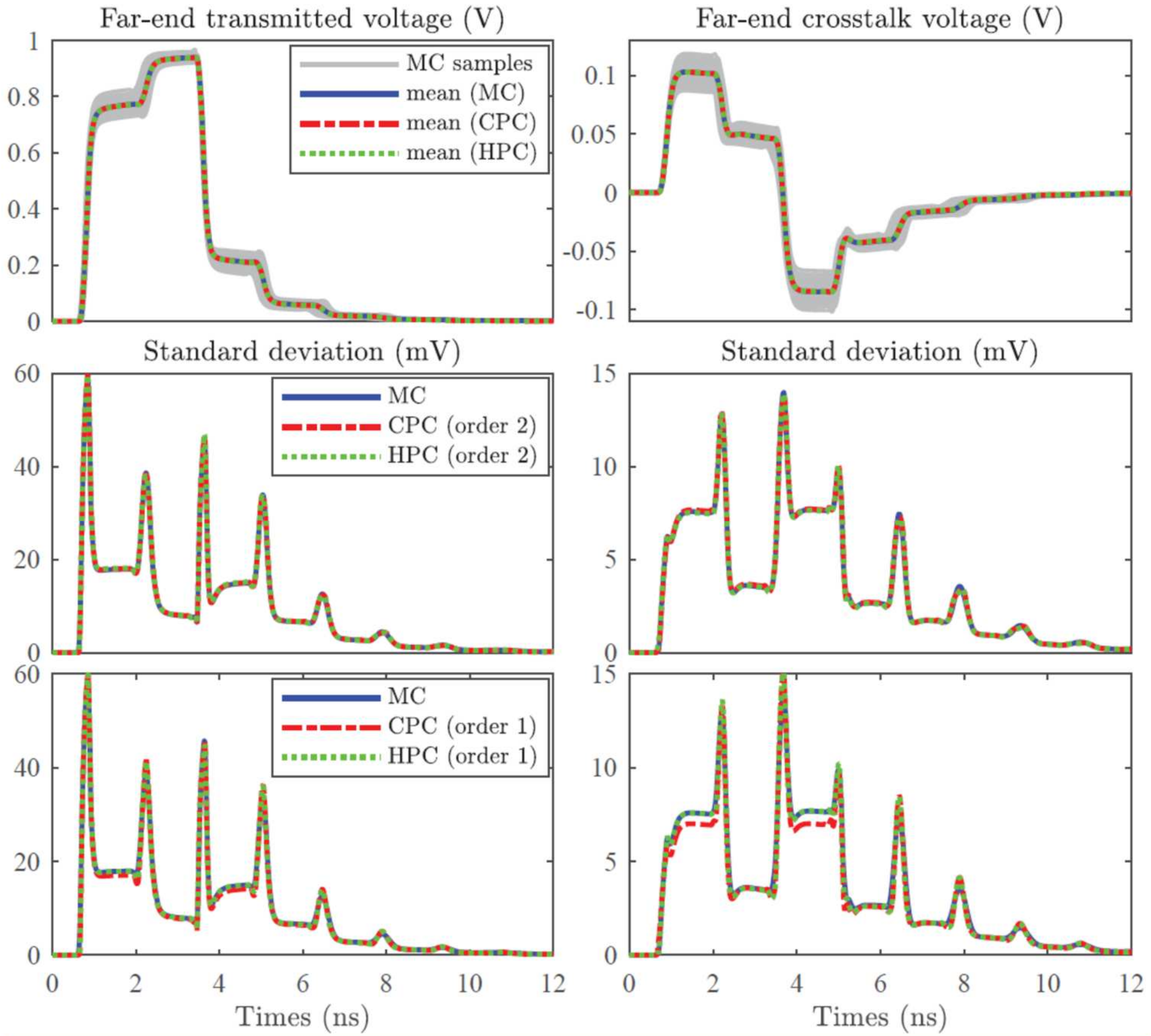
reciprocity

$$\boldsymbol{L}_{dc} = \begin{pmatrix} \overset{\eta_6}{\boxed{L_{11}}} & \overset{\eta_7}{\boxed{L_m}} \\ \boxed{L_m} & \underset{\eta_8}{\boxed{L_{22}}} \end{pmatrix}$$

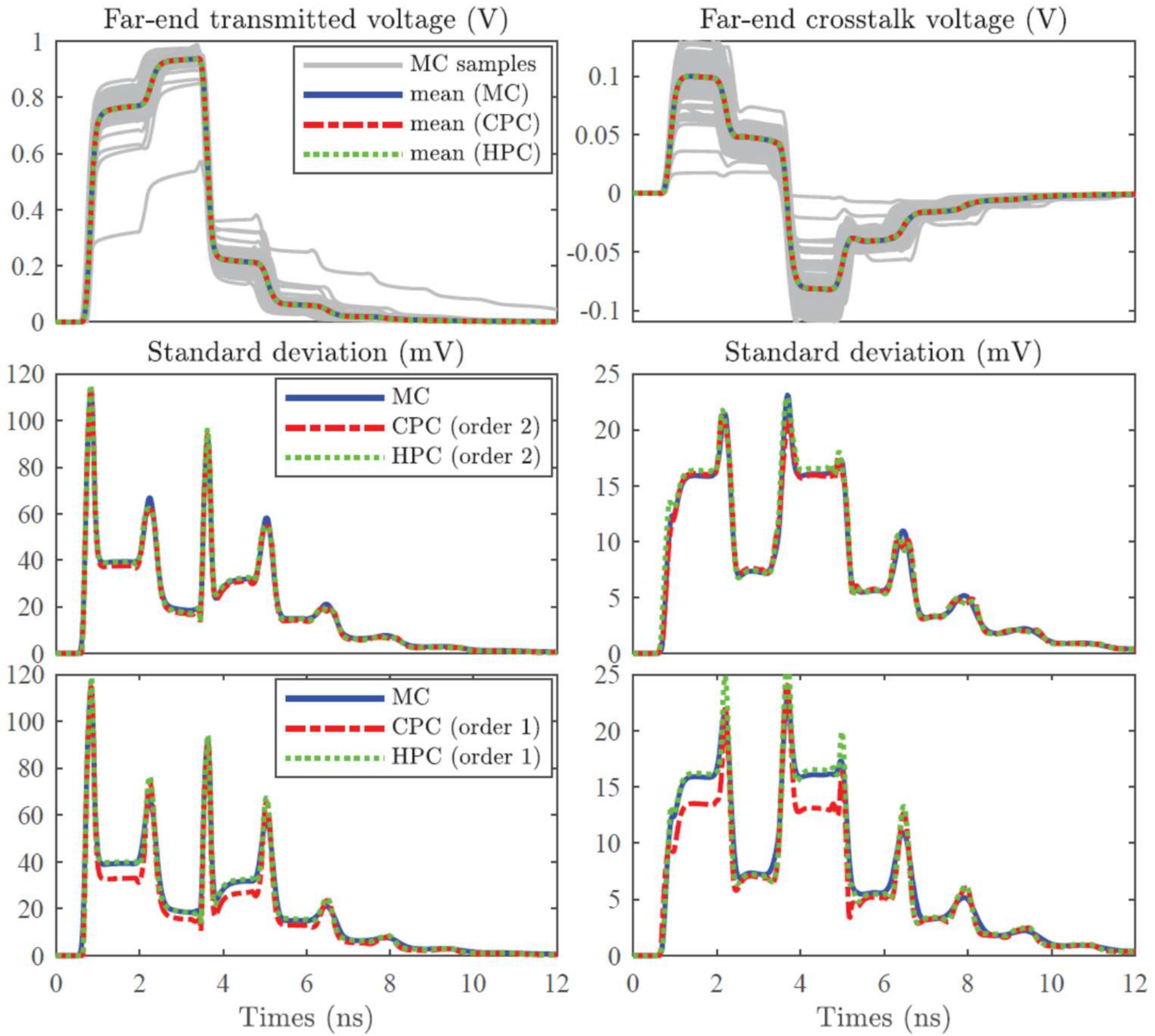
$$\boldsymbol{C}_{dc} = \underset{\eta_9}{\boxed{\varepsilon_r}}\varepsilon_0\mu_0\boldsymbol{L}_{dc}^{-1}$$

Homgeneous structure ⇒ C-matrix has explicit dependence on $\boldsymbol{L}_{dc}$!

Application #3: results (5% variation)



Application #3: results (10% variation)



Application #3: accuracy & efficiency

error definition:

$$\mathcal{E} = \frac{1}{T}\int_0^T \left|\mathrm{Std}_{\mathrm{PC}}(t) - \mathrm{Std}_{\mathrm{MC}}(t)\right| dt$$

variation	quantity	order 1		order 2	
		CPC	HPC	CPC	HPC
5%	far-end transmission	2.9915e-04	1.9635e-04	8.6582e-05	8.1490e-05
	near-end crosstalk	4.1755e-04	2.4029e-04	1.2283e-04	1.1526e-04
	far-end crosstalk	2.6472e-04	1.3034e-04	6.7092e-05	5.9144e-05
10%	far-end transmission	9.3660e-04	5.8507e-04	3.5322e-04	4.4294e-04
	near-end crosstalk	2.6854e-03	1.1444e-03	9.2660e-04	6.8755e-04
	far-end crosstalk	1.0048e-03	4.6998e-04	2.8028e-04	3.7999e-04

Basis functions	(hierarchical PC, order 1):	3.2 s	
Augmented matrices	(classical PC, order 2):	49.7 s	
	(hierarchical PC, order 1):	0.3 s	(165x)
SPICE simulation	(classical PC, order 2):	57.2 s	
	(hierarchical PC, order 1):	2.4 s	(24x)