A BAYESIAN APPROACH TO ADAPTIVE FREQUENCY SAMPLING

Simon De Ridder — simon.deridder@ugent.be
OVERVIEW

1  MOTIVATION

2  LINEAR BAYESIAN VECTOR FITTING

3  EXAMPLE
   - HAIRPIN FILTER

4  SUMMARY
MOTIVATION
THE NEED FOR ADAPTIVE FREQUENCY SAMPLING

- Characterization of devices through simulations is essential to design.

- Simulation at every frequency often too expensive.

- Need a broadband characterization with few simulations.
ADAPTIVE FREQUENCY SAMPLING

SEQUENTIAL STRATEGY

Classic approach: sweep over frequency range
Adaptive frequency sampling (AFS): Sequentially simulate at one frequency at a time. Support points → one or several macromodels → new simulation.
Adaptive Frequency Sampling
Sequential strategy

Classic approach: sweep over frequency range
Adaptive frequency sampling (AFS): Sequentially simulate at one frequency at a time. Support points → one or several macromodels → new simulation.
### Adaptive Frequency Sampling

**Sequential strategy**

**Classic** approach: sweep over frequency range

Adaptive frequency sampling (**AFS**): Sequentially simulate at one frequency at a time. Support points → one or several macromodels → new simulation.
**Adaptive Frequency Sampling**

**Sequential Strategy**

Classic approach: sweep over frequency range

Adaptive frequency sampling (AFS): Sequentially simulate at one frequency at a time. Support points → one or several macromodels → new simulation.
**Adaptive Frequency Sampling**

**Sequential Strategy**

**Classic** approach: sweep over frequency range

Adaptive frequency sampling (AFS): Sequentially simulate at one frequency at a time. Support points → one or several macromodels → new simulation.
Adaptive Frequency Sampling

Sequential strategy

Classic approach: sweep over frequency range
Adaptive frequency sampling (AFS): Sequentially simulate at one frequency at a time.

support points → one or several macromodels → new simulation.
**Adaptive Frequency Sampling**

**Sequential Strategy**

**Classic** approach: sweep over frequency range

Adaptive frequency sampling (**AFS**): Sequentially simulate at one frequency at a time. Support points $\rightarrow$ one or several macromodels $\rightarrow$ new simulation.
LINEAR BAYESIAN VECTOR FITTING
Goal of VF: modeling transfer function (e.g. S-parameters)
Approximate the transfer function with a rational pole/residue model

\[
\overline{F}(s) \approx \sum_{k=1}^{K} \frac{\overline{R}_k}{s - \overline{a}_k} + \overline{D} + \overline{sE}
\]

→ Nonlinear problem due to \( a_k \).
CLASSIC VECTOR FITTING

Starting poles $q_k$ → Identify residues of denominator → Calculate relocated poles → Identify residues $R_k$

Rewrite as

$$
\bar{F}(s) = \frac{\bar{p}(s)}{\sigma(s)} = \frac{\sum_{k=1}^{K} \frac{\hat{r}_k}{s - q_k} + \hat{d} + \bar{e}}{\sum_{k=1}^{K} \frac{\hat{r}_k}{s - q_k} + \hat{d}}
$$

Solve $\sigma(s)\bar{F}(s) = \bar{p}(s)$ for $\hat{r}_k$ and $\hat{d}$.

linear regression
CLASSIC VECTOR FITTING

Starting poles $q_k$ → Identify residues of denominator → Calculate relocated poles → Identify residues $\overline{R}_k$

\[
\overline{F}(s) = \frac{\overline{p}(s)}{\sigma(s)} = \frac{\sum_{k=1}^{K} \frac{\overline{r}_k}{s - q_k} + \overline{d} + s \overline{e}}{\sum_{k=1}^{K} \frac{\hat{r}_k}{s - q_k} + \hat{d}}
\]

Zeros of $\sigma(s)$ = poles of $\overline{F}(s)$.

(nonlinear) eigenvalue problem

$\rightarrow$ relocated poles $a_k$
CLASSIC VECTOR FITTING

Starting poles $q_k$ → Identify residues of denominator → Calculate relocated poles → Identify residues $R_k$

\[ \overline{F}(s) = \sum_{k=1}^{K} \frac{\overline{R_k}}{s - \overline{a_k}} + \overline{D} + s\overline{E} \]

Identify $\overline{R_k}$, $\overline{D}$ and $\overline{E}$.

linear regression
CLASSIC VECTOR FITTING

Starting poles \( q_k \)

Identify residues of denominator

Calculate relocated poles

Identify residues \( R_k \)

linear regression

eigenvalue problem

linear regression
Linear Bayesian Vector Fitting

1. Starting poles $q_k$
2. Identify residues of denominator
   - Bayesian linear regression
3. Calculate relocated poles
4. Identify residues $R_k$
5. Bayesian linear regression sampling
LINEAR BAYESIAN VECTOR FITTING

\[ q_k \rightarrow \hat{r}_k, \hat{d} \rightarrow a_k \rightarrow \bar{R}_k \rightarrow \text{LBVF model} \]

Sampling denominator residues
Calculating relocated poles
LINEAR BAYESIAN VECTOR FITTING

$q_k \rightarrow \hat{r}_k, \hat{d} \rightarrow a_k \rightarrow R_k \rightarrow \text{LBVF model}

\text{Sampling residues}
LINEAR BAYESIAN VECTOR FITTING

LBVF models
AFS with Linear Bayesian Vector Fitting

Initial frequency points → sample LBVF models → Calculate weighted uncertainty

Evaluate new frequency point

>threshold? yes → final rational model

no → >threshold?
AFS with Linear Bayesian Vector Fitting

- Initial frequency points
- Sample LBVF models
- Evaluate new frequency point
- Calculate weighted uncertainty
- > threshold?
- Final rational model

- Samples from LBVF models of different orders
- Weighted standard deviation using marginal likelihood as weights
- Gaussian penalties at already evaluated points
Example
HAIRPIN FILTER

| \(|S_{11}\) (dB)| Simulated data |
|----------------|---------------|
| 12.5 | 13.0 | 13.5 | 14.0 | 14.5 |
| 100  | 80   | 60   | 40   | 20   |

| \(|S_{21}\) (dB)| Simulated data |
|----------------|---------------|
| 12.5 | 13.0 | 13.5 | 14.0 | 14.5 |
| 16   | 30   | 15   | 8    | 5    |
HAI R P I N  F I L T E R

4 INITIAL POINTS

\[ S_{11} \]

\[ S_{21} \]

\( |S_{11}| \) (dB)

\( |S_{21}| \) (dB)
HAIRPIN FILTER

4 INITIAL POINTS

\[ S_{11} \]

\[ S_{21} \]

-60  -50  -40  -30  -20  -10  0  10

12.5  13.0  13.5  14.0  14.5

Frequency (GHz)

\(|S_{11}| \) (dB)

\(|S_{21}| \) (dB)

1 pole samples
2 pole samples
3 pole samples
known points
HAIRPIN FILTER
4 INITIAL POINTS

$S_{11}$

$S_{21}$
HAIRPIN FILTER
5 POINTS

$S_{11}$

$S_{21}$
Hairpin Filter

5 Points

$S_{11}$

$S_{21}$
HAIRPIN FILTER

5 POINTS

$S_{11}$

$S_{21}$
HAIRPIN FILTER

6 POINTS

\[ S_{11} \]

\[ S_{21} \]

Uncertainty

<table>
<thead>
<tr>
<th>( S_{11} ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 pole samples</td>
</tr>
<tr>
<td>3 pole samples</td>
</tr>
<tr>
<td>4 pole samples</td>
</tr>
<tr>
<td>5 pole samples</td>
</tr>
<tr>
<td>known points</td>
</tr>
<tr>
<td>Next point</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( S_{21} ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 pole samples</td>
</tr>
<tr>
<td>3 pole samples</td>
</tr>
<tr>
<td>4 pole samples</td>
</tr>
<tr>
<td>5 pole samples</td>
</tr>
<tr>
<td>known points</td>
</tr>
<tr>
<td>Next point</td>
</tr>
</tbody>
</table>
HAIRPIN FILTER
7 POINTS

$S_{11}$

$S_{21}$
HAIRPIN FILTER

8 POINTS

$S_{11}$

$S_{21}$
HAIRPIN FILTER

9 POINTS

\[ S_{11} \]

\[ S_{21} \]
HAIRPIN FILTER
10 POINTS

$S_{11}$

$S_{21}$
**HAIRPIN FILTER**

**11 POINTS**

![Graphs of S11 and S21](image)

- Frequency (GHz)
- |S11| (dB)
- |S21| (dB)
- 7 pole samples
- 8 pole samples
- 9 pole samples
- 10 pole samples
- Known points
- Next point

Uncertainty: $1 \times 10^{-2}$
HAIRPIN FILTER

BEST MEAN FIT AT EACH STEP

**S\textsubscript{11}**

- best mean fit
- known points

**S\textsubscript{21}**

- best mean fit
- known points
**HARPin FILTER**

**FINAL MEAN FIT**

![Graphs showing |S11| and |S21| for different frequencies. The graphs display the simulated data, best fit, and evaluated points.]
SUMMARY

- LBVF is a next-generation stochastic modeling framework based on Vector Fitting.

- It provides a useful measure of model uncertainty.

- Key advantages:
  - provides model uncertainty in a principled and statistically sound manner
  - can handle noisy (non-deterministic) data
A BAYESIAN APPROACH TO ADAPTIVE FREQUENCY SAMPLING

Simon De Ridder — simon.deridder@ugent.be
HAIRPIN FILTER
UNCERTAINTY QUANTIFICATION WITH GAUSSIAN NOISE

- Relocated poles
- Relocated pole

\[
\frac{(3 - 85.0307) \times 10^{-10}}{(\Re + 0.346671) \times 10^{-10}}
\]
HAIRPIN FILTER

UNCERTAINTY QUANTIFICATION WITH GAUSSIAN NOISE

![Graphs showing S11 magnitude and phase vs. frequency with Gaussian noise uncertainty.]
DOUBLE SEMI-CIRCULAR PATCH ANTENNA

1590 µm $\varepsilon_r = 2.62$

1590 µm $\varepsilon_r = 2.62$
DOUBLE SEMI-CIRCULAR PATCH ANTENNA

AFS
DOUBLE SEMI-CIRCULAR PATCH ANTENNA
AFS
DOUBLE SEMI-CIRCULAR PATCH ANTENNA
AFS

![Graph showing frequency response with uncertainty]
DOUBLE SEMI-CIRCULAR PATCH ANTENNA AFS

**Graph:**
- **Uncertainty:** 1e-3
- **Magnitude (dB):**
  - 4 pole samples
  - 5 pole samples
  - 6 pole samples
  - Known points
  - Next point

**Frequency (GHz):**
- 2.0
- 2.5
- 3.0
- 3.5
- 4.0

**Magnitude (dB):**
- 0
- -20
- -40
- -60
DOUBLE SEMI-CIRCULAR PATCH ANTENNA

AFS

![Graph of double semi-circular patch antenna characteristics](image-url)

The graph above illustrates the performance of a double semi-circular patch antenna (AFS) across different frequencies. The graph shows the magnitude (dB) response at various frequencies ranging from 2.0 GHz to 4.0 GHz.

Key features:
- Five different pole samples are plotted, each represented by a different line color.
- The graph includes known points and a next point marker.
- Uncertainty is indicated by a range around the plotted points.

The figure provides a clear visualization of how the antenna's performance changes with frequency, highlighting its characteristics at various points in its frequency response.
DOUBLE SEMI-CIRCULAR PATCH ANTENNA
AFS
DOUBLE SEMI-CIRCULAR PATCH ANTENNA

AFS

![Graph showing frequency response and uncertainty for a double semi-circular patch antenna. The graph displays the magnitude in dB and uncertainty across different frequencies. The frequency range is from 2.0 to 4.0 GHz, and the magnitude varies from -40 dB to 0 dB. The graph includes markers for known points and the next point, with uncertainty indicated by a 1e-3 scale.]
DOUBLE SEMI-CIRCULAR PATCH ANTENNA

AFS

![Graph showing frequency and magnitude for different pole samples and known points.](image)
DOUBLE SEMI-CIRCULAR PATCH ANTENNA

AFS